Urban Studies

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Housing bubble contagion from city centre to suburbs

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Abstract

Most previous studies have reported that housing prices diffuse from the city centre to surrounding areas. However, these studies have overlooked the fact that housing prices comprise fundamental and bubble prices. We investigated whether bubble prices also diffuse from the city centre to suburbs and whether fundamental or bubble prices promote housing price diffusion. We focused on the movement of housing bubbles from the city centre to the suburbs. Using data for the Taipei metropolitan area from 1973 to 2014 for empirical analysis, our state-space model estimates statistically significant fundamental and bubble prices in Taipei City (city centre) and New Taipei City (suburbs). Engle–Granger cointegration test results reveal that the housing and bubble prices of the two cities are cointegrated; however, fundamental prices are not. F statistics reveal that the Granger causality of bubble prices (the central city Granger causing changes in the suburbs) is more significant and powerful than that of the fundamental prices. Therefore, we demonstrate that housing bubbles force housing price diffusion. In addition, when bubble prices spread from the city centre to the suburbs, the housing bubble in the suburbs is larger than that in the city centre, implying that the suburbs have greater potential for a bubble burst crisis called the bubble contagion. Authorities should pay more attention to the bubble contagion and must address the problem of high housing prices in the suburbs to prevent this bubble from bursting

Introduction

The Taipei metropolitan area (Taipei City and New Taipei City together) has a

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Figure 1. Geographical relationship between Taipei City and New Taipei City.

population of 6.5 million, accounting for approximately one-third of the population of Taiwan. Figure 1 depicts the Taipei metropolitan area, with New Taipei City surrounding Taipei City. Geographically, Taipei City and New Taipei City have a concentric circle layout. Taipei City functions as the city centre and New Taipei City as its suburb; therefore, the Taipei metropolitan area offers a suitable opportunity to investigate the change in housing prices between the city centre and the suburbs.

Several previous studies have reported that housing prices diffuse from the city centre to the suburbs (Berg, 2002; Meen, 1996; Oikarinen, 2006), and Chen et al. (2011) have reported a bidirectional relationship between Taipei City and New Taipei City. However, previous studies (Alessandri, 2006; Blanchard and Fisher, 1989; Fraser et al., 2008; Xiao and Tan, 2007)[1] have failed to provide insight on whether price diffusion originates from fundamental or

bubble prices. In other words, housing price diffusion factors may originate from the fundamental or bubble price. The fundamental price is that based on living demands, whereas the bubble price reflects speculative behaviours. For example, when employment opportunities increase in suburbs and people migrate from the city to the suburbs, the demand for owner-occupied home purchases increases in suburbs, stimulating a rise in suburban housing prices. In this case, housing price diffusion is caused by the fundamental rather than bubble price. Conversely, when people migrate from the city to the suburbs according to investment behaviour, they buy houses for short-term trading rather than residing, further stimulating the price rise in suburban housing. In this case, housing price diffusion is caused by the bubble rather than fundamental price. Therefore, exploring whether fundamental or bubble prices promote housing price diffusion is worthwhile. Most studies have indicated that Taipei City has had a housing bubble since 2006 (Chang et al., 2009; Teng et al., 2013; Tsai and Peng, 2011; Wang et al., 2011). However, it remains unclear whether a housing bubble contagion from Taipei City will bring about a housing bubble in New Taipei City and, if the housing bubble spreads, whether it is larger in the city or the suburbs. Because the influence of the housing bubble contagion is extremely infrequently studied, this study examined the movement of a housing bubble from the city to the suburbs of the Taipei metropolitan area in Taiwan.

The housing market in Taiwan boomed after the SARS outbreak in 2002 and 2003, and housing prices began to rise in 2004. However, the rise in housing prices occurred only in the Taipei metropolitan area and was not widespread throughout Taiwan. Figures 2 and 3 show that from the first quarter of 2004 to the fourth quarter of 2014, the housing price growth rates were
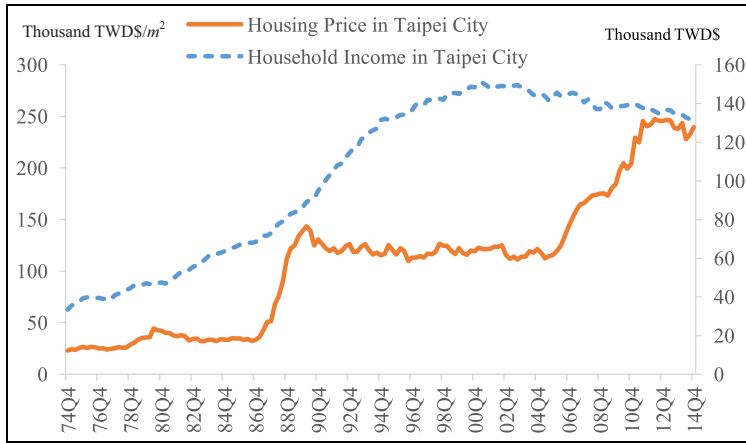
Figure 2. Taipei city housing price and household income from 1973Q2 to 2014Q4.
Source: Housing price: Taiwan Real Estate Research Center, National Chengchi University; household income: Directorate-General of Budget, Accounting and Statistics, Executive Yuan, R.O.C.
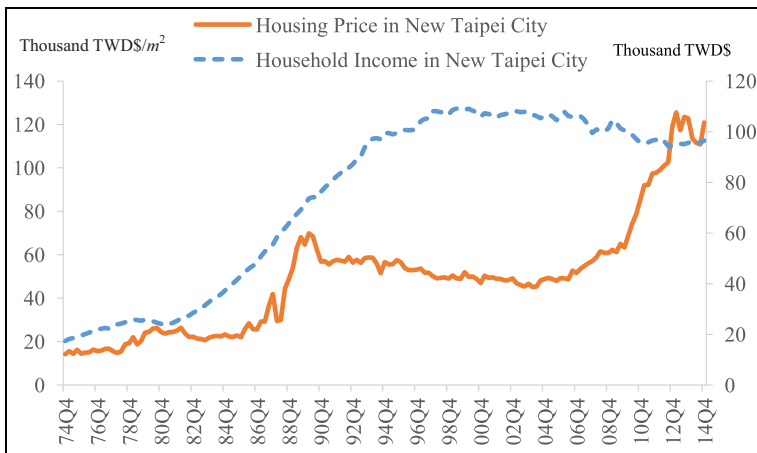
Figure 3. New Taipei City housing price and household income from 1973Q2 to 2014Q4.
Source: Housing price: Taiwan Real Estate Research Center, National Chengchi University; household income: Directorate-General of Budget, Accounting and Statistics, Executive Yuan, R.O.C.

110% and 168% in Taipei City and New Taipei City, respectively. However, simultaneously, household income decreased in both cities. Clearly, the housing prices in the Taipei metropolitan area are an increasing burden on the residents.

Riddel (2011) reported that housing investment in peripheral markets is a substitute for investment in the core of the urban market. Investors searching for additional investment opportunities often find housing prices in the peripheral market lower than those in the urban core. In addition, compared with the city centre, suburbs have more opportunities for future development. These factors attract speculative

demand in the suburban housing market. Furthermore, according to the National Population Census of 2010, the vacancy rate in suburbs was 22% higher than that in the centre was 12%. Clearly, New Taipei City is experiencing a more serious problem of high housing prices and vacancy rates compared with Taipei City. Therefore, we believe that the housing bubble is likely to spread from Taipei City to New Taipei City and create a housing price ripple effect. However, we expect that the scale of the initial housing bubble in New Taipei City is smaller than that in Taipei City, and with an increase in speculative behaviours in the suburbs, the housing bubble of New Taipei City may be greater than that of Taipei City; we call this phenomenon the 'bubble contagion'.

This study examined the diffusion in housing price, fundamental and bubble prices and the differences in housing bubbles between Taipei City as the central market and New Taipei City as its peripheral market. First, we defined a housing bubble as a deviation in housing prices from its fundamental value according to household income. We used a state-space model to empirically estimate the housing bubbles in Taipei City and New Taipei City since the 1970s. To achieve our objective, we used the Engle–Granger cointegration and Granger causality tests to examine the interaction of housing fundamental and bubble prices in terms of contagion and diffusion from the city to the suburbs. In addition, we compared the bubble sizes between regions. We found that there are housing bubbles in the Taipei metropolitan area, and the bubble contagion is the main factor pushing housing price diffusion from the city centre to the suburbs. Our contribution is the clarification of the bubble contagion and housing price diffusion. Specifically, although high housing prices diffuse from the city centre, the bubble in the suburbs is larger than that in the city centre. Therefore, the government must address the problem of high housing prices in the suburbs to prevent this bubble from bursting.

Literature review

Previous studies have evidenced that housing price shock in one area is likely to be experienced in other areas (Alexander and Barrow, 1994; Ashworth and Parker, 1997; Chen et al., 2011; Holly et al., 2011; Pollakowski and Ray, 1997; Stevenson, 2004; Vansteenkiste and Hiebert, 2011). Meen (1999) used the augmented Dickey-Fuller test to examine housing prices in Britain and observed that prices rose first in a cyclical upswing in the southeast, and then spread over the rest of the country. This is known as 'housing price diffusion' or the 'ripple effect'. Regarding housing price diffusion, Meen (1999) believed that convergence exists if long-run equilibrium relationships occur between regional housing markets. Cook (2003) concluded that convergence rather than disparities in the real price explains why regional housing prices return to an equilibrium. Despite differing opinions on housing price diffusion, since Meen (1996) examined regional housing price dynamics, most previous studies have assumed that housing price movements diffuse from the economic centre to the surrounding regions. The leading role of the main economic centre has often been based on the assumption that business cycles affect economic centres first and peripheral areas later.

In addition, several factors, such as migration, structural differences in regional markets, economic interdependence, informational factors and the wealth effect of housing, explain why housing price changes in central areas may lead to housing price movements in surrounding regions (Benjamin et al., 2004; Gabriel et al., 1999; Jones and Leishman, 2006; Meen, 1999; Oikarinen, 2006). Most studies agree that growing demand for houses is the leading

factor affecting housing price diffusion. Gabriel et al. (1999) applied insights from economic theory to explain recent housing price patterns in the two largest metropolitan areas in California. They found that migration between metropolitan areas is crucial in explaining the overall housing price dynamics for a given metropolitan area. Furthermore, long-run persistent housing price differentials can be explained partly by the distribution of housing quality and amenities within particular areas. However, living and investment demands critically govern the growing demand for houses. Growth in demand originates primarily from migration, as well as an increase in income and employment, contributing to an increase in living demand and thus stimulating an increase in the fundamental housing price. In this situation, housing prices diffuse, but bubble prices do not.

Conversely, Acemoglu (1995) stated that agents choose activities yielding the highest private returns. The goal of investors is to reduce investment costs to obtain the highest returns; therefore, they invest in peripheral markets, and their search for housing is similar to rent seeking. Furthermore, Murphy et al. (1993) reported that an increase in rent-seeking activity may increase the attractiveness of rent seeking relative to productive activity, and rent seeking is likely to harm innovative activities more than everyday production. We consider that in the housing market, housing consumers increase the robustness of the housing market. Investors or speculators are similar to rent seekers in that they look for personal benefits and make no contribution to the housing market. For example, if investors expect the housing price in the city centre and suburban house markets to converge either in the absolute price or rate of appreciation and buy numerous houses in the suburbs only for investment, then the growing demand in peripheral areas is based on the expectation

of future price appreciation. In this situation, housing prices increase from the city centre to its surrounding regions, resulting from the investors' belief that the sale price will be higher in the future, although the fundamental factors do not justify such a price (Stiglitz, 1990). Therefore, we propose that the bubble price, but not the fundamental price, accelerates increases in the housing price in areas surrounding the city centre. Riddel (2011) developed a theoretical model that accounts for speculative price appreciation spreading from one market to another and used an error correction model to determine whether contagious prices and income growth from Los Angeles sustained by naive expectations contributed to a bubble in Las Vegas, Nevada. However, that paper lacked a clear definition of fundamental and bubble prices. Moreover, the housing price in the city must be higher than that in the suburbs if the housing bubble is contagious and the city centre housing bubble is greater than that of the suburbs. The influence of the housing bubble contagion has been ignored and is worth exploring.

The housing bubble is an unobservable variable; hence, according to Flood and Hodrick (1986), bubble tests are difficult to design because the path of a bubble in the data appears as an error in the modelling of agents' expectations. The most commonly used approach is to benchmark housing prices against equilibrium prices predicted by economic models (Case and Shiller, 2003; Hui and Shen, 2006; Smith et al., 2006). However, the problem of this approach is that the economic models are prone to misspecification. To overcome these problems, Wu (1995) employed the Kalman filter technique to estimate and test stochastic bubbles in foreign exchange markets, showing that stock-price deviations from the present-value model are recaptured as bubbles in his model. Bertus and Stanhouse (2001) employed state-space model and dynamic

factor analysis to document the price bubble in gold futures markets. Recent studies have used the state-space model to test for the existence of bubbles in housing or stock markets (Alessandri, 2006; Lau et al., 2005; Teng et al., 2013; Wu, 1997; Xiao and Park, 2010; Xiao and Tan, 2007). Therefore, in this study, we used the state-space model to calculate the bubble price.

Numerous studies have evaluated bubble in the housing market (Abraham and Hendershott, 1996; Ansgar and Marcel, 2005; Beck et al., 2012; Case and Shiller, 2003; Donald, 2006; Flood and Hodrick, 1986; Gaia and Lucio, 2004; McMillan and Speight, 2010; Smith et al., 2006; Wheaton and Nechayev, 2008) or the monetary factors that influence housing bubbles (Bredthauer and Geppert, 2013; Iqbal and Vitner, 2013; Kohn and Bryant, 2011; McDonald and Stokes, 2013; Miles, 2014; Morrow, 2012). All these studies have used different methodologies to research and analysis regions or cities with housing bubbles. However, they have not compared the bubble sizes in different regions and explained these differences. Only Teng et al. (2013) examined the effects of different land tenure systems on the size of housing bubbles. They reported that a housing bubble is likely to be larger in a market with freehold properties than in a market with leasehold properties. However, no study has evaluated the bubble contagion thus far. To fill this gap, we focused on housing prices, fundamental prices, and bubble price diffusion and explored the influence of the bubble contagion. This is the first study to separate the housing price into fundamental and bubble prices to examine housing price diffusion.

Methodology

Fundamental housing price–income model

People who buy housing for living consider buying only when the housing price is within the range that they can afford. Capozza et al. (2004) and Case and Shiller (2003) have reported that real income and interest rates are critical factors contributing to the real housing price. Furthermore, a long-term equilibrium relationship exists between income and housing prices in Taiwan (Chen and Patel, 1998; Chen et al., 2007; Tsai and Peng, 2012). Chang et al. (2009) stated that both rent and income are suitable indices for measuring the fundamental price in Taipei City. The rent data in New Taipei City is limited; therefore, in this study, we used income to measure the fundamental price. We adopted the method of Black et al. (2006), in which the expected value of future real disposable income is treated as fundamental before converting it to a state-space model for calculating the bubble price. The model is shown as follows:

$$P_t = E_t \sum_{i=1}^{\infty} \left(\frac{1}{\prod\limits_{j=1}^{i} (1 + \rho_{t+j})} \right) Q_{t+i} \quad (1)$$

where P_t is the real price at the end of period t; Q_{t+1} is the real permanent income measure during $t + 1$; and ρ is the discount rate. Equation (1) is a particular solution to $P_t = E(P_{t+1} + Q_{t+1})/(1 + \rho_{t+1})$. By taking the log and using lowercase letters to represent the logs of their uppercase counterparts, we have $r_{t+1} \equiv ln(1 + \rho) = ln(1 + exp(q_{t+1} - p_{t+1})) + p_{t+1} - p_t$, where the term $(q - p)$ can be viewed as the income-to-price ratio. By applying the first-order Taylor expansion, we obtain Equation (2):

$$r_{t+1} = -(p_t - q_t) + \mu(p_{t+1} - q_{t+1}) + \Delta q_{t+1} + k \quad (2)$$

where k and μ are linearisation constants; specifically, $0 < \mu < 1$ and in practice is close to 1. The constants are expressed as follows:

$$k = -\ln \mu - (1 - \mu)\overline{(q - p)}$$

$$\mu = 1 / \left(1 + \exp\overline{(q - p)}\right)$$

where : $\overline{(q - p)}$ is the mean sample of $(q - p)$.

Denote the (log) price-income ratio, $(p_t - q_t)$, as pq_t and rearrange Equation (2) as follows:

$$pq_t = k + \mu pq_{t+1} + \Delta q_{t+1} - r_{t+1} \qquad (3)$$

After repeated substitutions in Equation (3), let $t \to \infty$, assume that the limit of the final term is 0, and take conditional expectations. Using the method of Black et al. (2006), we arrive at the following expression for the empirical model, described in detail in the Appendix:

$$pq_t = \frac{k - f}{(1 - \mu)} + \sum_{j=0}^{\infty} \mu^{j+1} E_t \Delta q_{t+j+1}$$
$$- \alpha \sum_{j=0}^{\infty} \mu^{j+1} E_t \sigma_{t+j+1}^2 \qquad (4)$$

In order to use Equation (4) to generate a series for pq_t^* and p_t^*, according to Black et al. (2006), we use a three-variable vector autoregression (VAR) $(pq_t, \Delta q_t, \text{and } \sigma_t^2)$ to forecast real permanent income growth and housing return variance and then measure the ratio of the fundamental housing price to permanent income, pq_t^*. Finally, the housing fundamental can be generated as:

$$p_t^* = pq_t^* + q_t \qquad (5)$$

where p_t^* is the (log) fundamental measure of housing price. At any time point, the bubble price (b_t) (if it exists) is defined as a deviation of the market price (ignoring any measurement error) from the fundamental price, which is an unobservable variable. We assume that the bubble price is a stochastic froth, follow the AR(1) process, and use the state-space model for measuring the housing bubble.

$$p_t = c_1 pq_t^* + c_2 q_t + b_t + v_t \qquad (6)$$

$$b_t = \psi^* b_{t-1} + \varpi_t \qquad (7)$$

where : $E(v_t \omega_s') = 0$, $E[\varpi_t] = 0$ $Var[\varpi_t] = \sigma_\varpi^2$

Because p_t has an unobservable component, b_t, standard regression procedures cannot be used to estimate the parameters of Equation (6). A state-space model is a time series model that includes one or more unobservable variables, the dynamics of which can be represented using a state equation. The parameters of the observation and state equations can be jointly estimated using maximum likelihood methods. Estimating Equation (6) (called the observation or measurement equation) entails considering the dynamics of the unobservable variable (called the state variable) represented by Equation (7), which is also called the state equation. Equations (6) and (7) together constitute a state-space model. The state-space model enables separating the deviation of the observed market price from the fundamental price and representing it as measurement error (v_t) by using a white noise process and the bubble price (b_t).

Cointegration test with regional fundamental, bubble and housing prices

Many studies on asset price diffusion have employed various methodologies to identify relationships between variables. Alexander and Barrow (1994), Chen et al. (2011), Chien (2010), Cook (2003), MacDonald and Taylor (1993), Oikarinen (2006), Stevenson (2004), and Vansteenkiste and Hiebert (2011) have reported that housing price changes in one area spread to other areas. MacDonald and Taylor (1993) and Alexander and Barrow (1994) have analysed relationships between regional housing markets by using the Engle-Granger

cointegration and Granger causality tests. Stevenson (2004) and Oikarinen (2006) used the Johansen cointegration test and a vector-error correction model (VECM) to examine housing price diffusion in the Republic of Ireland and Finland, respectively. Chen et al. (2011) examined dynamic links amongst four regional house price indices in Taiwan and used the Johansen cointegration technique, the Granger causality test, the generalised impulse response approach, and various decomposition analyses to identify the extent and magnitude of relationships amongst the indices.

Similar to previous studies, we used the Engle-Granger cointegration and Granger causality test for testing the diffusion of overall housing price, fundamental, and bubble prices in Taipei City and New Taipei City. We first perform an ordinary least squares (OLS) regression for the two test cities.

$$P_{k,t}^{\theta} = \widehat{P_{k,t}^{\theta}} + e_{k,t}^{\theta} = \alpha_{k,0}^{\theta} + \alpha_{k,1}^{\theta} P_{k',t}^{\theta} + e_{k,t}^{\theta} \quad (8)$$

where $P_{k,t}^{\theta} = \left(P_{k,t}^{h}, P_{k,t}^{f}, P_{k,t}^{b} \right)$, $P_{k,t}^{h}$ is the housing price, $P_{k,t}^{f}$ is the fundamental price, and $P_{k,t}^{b}$ is the bubble price. When $k = c$ (Taipei City as the central market), $k' = s$ (New Taipei City as the suburbs). After the OLS regression of $P_{k,t}^{\theta}$ on $P_{k',t}^{\theta}$, we obtain the residual through Equation (9). The unit root test is used to test whether $e_{k,t}^{\theta}$ is stationary. If $e_{k,t}^{\theta}$ is stationary, then $P_{k,t}^{\theta}$ and $P_{k',t}^{\theta}$ are cointegrated.

$$e_{k,t}^{\theta} = P_{k,t}^{\theta} - \widehat{P_{k,t}^{\theta}} \qquad (9)$$

According to the Granger representation theorem, if cointegration exists within a variable, then the direction of causality within the variable can be identified using an error correction model. In a two I(1) variable model, the error correction model is constructed using Equation (10).

$$\Delta P_{k,t}^{\theta} = \beta_0 + \beta_1 \Delta P_{k',t}^{\theta} - \lambda ecm_{t-1} + \mu_t \quad (10)$$

The preceding equation is a single equation of ECM, which can be used in bivariate (Taipei City and New Taipei City) systems comprising a VECM. If we assume that $P_{k,t}^{\theta}$ and $P_{k',t}^{\theta}$ affect each other with distributed lags, the relationship between the variables can be captured by a bivariate VECM. The Granger representation theorem states that, if the previous values of $P_{k,t}^{\theta}$ significantly contribute in forecasting the value of $P_{k',t}^{\theta}$, then $P_{k,t}^{\theta}$ can be considered to Granger cause $P_{k',t}^{\theta}$ and vice versa. This Granger causality specification is based on variables that are cointegrated.

Data description

This study focused on the diffusion of housing prices, fundamental, and bubble prices from the city centre (Taipei City) to the suburbs (New Taipei City) from the second quarter of 1973 to the fourth quarter of 2014. Quarterly housing price data were obtained from the Taiwan Real Estate Research Center, National Chengchi University. Data for household income were downloaded from the online system of the Directorate-General of Budget, Accounting and Statistics, Executive Yuan, R.O.C.[2] In this study, household current income was converted to permanent income.[3] All raw data are in nominal terms, which were deflated by respective consumer price indices to obtain the real price and real income data.

Table 1 lists descriptive statistics for the Taipei metropolitan area. In this table, we divided the full period (Part A) of the time series data into the three decades, the second quarter of 1973 to the fourth quarter of 2004 (Part B), the first quarter of 2005 to the

Table 1. Descriptive statistics of variables in Taipei and New Taipei, 1973Q2 to 2014Q4.

	DLTPHP	DLNTPHP	DLTPIC	DLNTPIC	UR
Part A:1973Q2~2014Q4					
Mean	0.0129	0.0123	0.0077	0.0086	6.3647
Std. dev.	0.0596	0.0735	0.0193	0.0241	3.8118
Skewness	1.1014	0.4250	−2.3407	−2.6530	0.3517
Kurtosis	7.0098	10.2663	20.9157	20.1941	2.1933
Part B: 1973Q2-2004Q4					
Mean	0.0116	0.0090	0.0108	0.0121	7.8401
Std. dev.	0.0653	0.0804	0.0205	0.0258	3.1648
Skewness	1.1291	0.4979	−2.9313	−3.1412	0.3471
Kurtosis	6.4073	9.4085	23.4541	21.6787	2.5208
Part C: 2005Q1-2009Q4					
Mean	0.0208	0.0131	−0.0025	−0.0027	2.0293
Std. dev.	0.0302	0.0284	0.0131	0.0155	0.3717
Skewness	−0.2939	0.6006	0.5108	0.3363	0.0090
Kurtosis	2.7049	2.6408	2.4294	2.5175	1.8744
Part D: 2010Q1-2014Q4					
Mean	0.0132	0.0323	−0.0018	−0.0020	1.4048
Std. dev.	0.0424	0.0550	0.0068	0.0089	0.1679
Skewness	0.7031	−0.1314	0.0510	−0.6052	0.2175
Kurtosis	3.5916	2.8677	1.5821	2.0730	1.8168

Note: DLTPHP is the rate of change in Taipei City housing prices; DLNTHP is the rate of change in New Taipei City housing prices; DLTPIC is the rate of change in Taipei City household permanent incomes; DLNTPIC is the rate of change in New Taipei City household permanent incomes; and UR is the risk-free rate.

fourth quarter of 2009 (Part C), and the first quarter of 2010 to the fourth quarter of 2014 (Part D). Overall, in Part A, the rate of change of the mean housing price remained almost the same in the city centre and the suburbs. The rates of change of household permanent income in the city centre and suburbs are positive. Part B shows that the rate of change in the mean housing prices in the suburbs is close to that in the city centre, and the rates of housing price and household permanent income change increase in the city centre and suburbs. However, when the housing market began to grow after 2004 (Part C), the rate of housing price change in city centre was higher than that in the suburbs. During this period, the household permanent incomes in the city centre and suburbs decreased. In Part D, the rate of housing price change in the suburbs surpasses that in the city centre,

implying that housing prices diffused from the city centre to the suburbs. Table 2 shows the results of the augmented Dickey-Fuller unit roots test, indicating that the housing price and income are I(1) stable series.

Analysis of bubbles in Taipei City and New Taipei City

The results of estimating the state-space model for Taipei City and New Taipei City are listed in Table 3. We found that c_2 is positive in both Taipei City and New Taipei City, representing a positive relationship between the housing price and household income. In addition, ψ is positive and significant in both cities, indicating clear bubble price fluctuations in Taipei City and New Taipei City. Thus, Taipei City and New Taipei City have housing bubbles.

Table 2. Augmented Dickey-Fuller test results.

Index	Level	Difference
Taipei housing price[a]	−2.62	−5.35***
New Taipei housing price[a]	−1.92	−3.86***
Taipei household income[b]	−1.22	−3.62***
New Taipei household income[b]	−0.42	−2.60**

Notes: [a]The constant was included in the test for the level of this index. [b]The constant and trend were not included in the test for the level of this index. ** significant at 5% level; *** significant at 1% level.
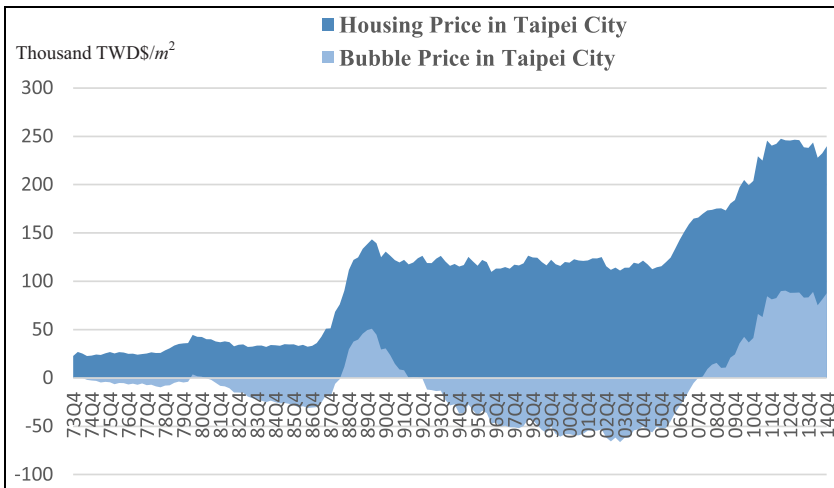
Table 3. State-space model estimated using the fundamental price according to income in Taipei City and New Taipei City.

$$p_t = c_1 pq_t^* + c_2 q_t + b_t + v_t$$
$$b_t = \psi b_{t-1} + \varpi_t$$

	c_1	c_2	ψ
Centre (Taipei City)	−1.07(0.91)	1.32(0.17)***	0.99(0.01)***
Suburbs (New Taipei City)	0.34(0.79)	1.02(0.13)***	0.98(0.02)***

*Note*: Numbers in parentheses are standard deviations. *** significant at 1% level.

Figure 4. Housing price and bubble price trends in Taipei City.

Figures 4 and 5 show the trends of the housing and bubble prices deduced from the state-space models of Taipei City and New Taipei City. For this model, the results show that both bubbles are statistically significant in the two cities from 1988 to 1992 and from 2007 to 2014.

The main cause of the increase in the housing price from 1987 to 1990 was a rapid expansion caused by a continuous increase
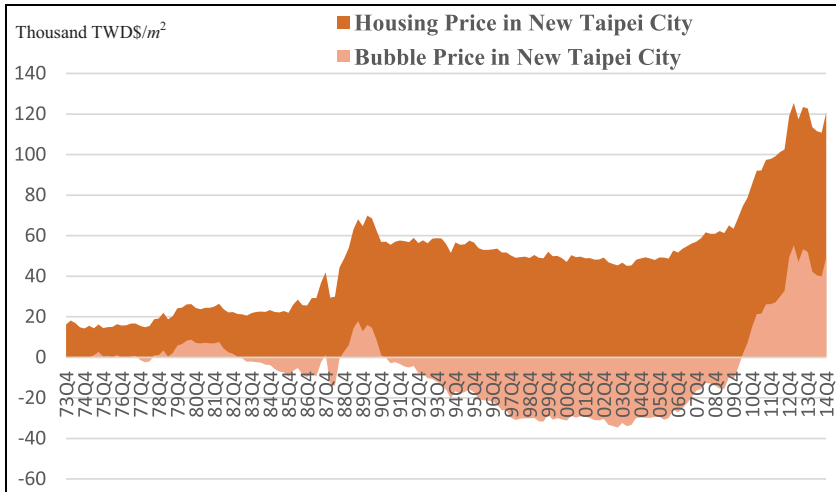
Figure 5. Housing price and bubble price trends in New Taipei City.

in the money supply (Chen et al., 2012). Because of financial liberalisation and loose monetary policies, a rapid economic growth rate of approximately 12% to 13% was observed. In 1990, the bubble accounted for 30% of the housing price in the city and 21% in the suburbs. Beginning in 1989, the government proposed policies such as raising interest rates and instituting loan controls to reduce the money supply. The Taiwanese central bank adopted selected credit control measures, including stopping uncollateralised loans for land, restricting the number of construction loans, and holding loan-to-value ratios below 50%. These measures created a recession in the economy and weakened the housing market considerably.

Since 2004, the real estate market in Taiwan has grown, and housing prices have increased dramatically, particularly in Taipei City and New Taipei City. This upward trend, which has continued to the present, is the longest of its type in many years and has caused a heavy burden on ordinary home buyers. Furthermore, this trend may continue into the near future.

Figure 6 shows estimates of the housing bubbles. In the fourth quarter of 2007, a housing bubble appeared in the city centre. After the second quarter of 2010, a housing bubble appeared in the suburbs. From the third quarter of 2007 to the third quarter of 2012, the housing bubble in the city centre was larger than that in the suburbs. However, after the fourth quarter of 2012, the housing bubble in the suburbs was larger than that in the city centre. This phenomenon is unprecedented, although the average difference was 5% for over 2 years. According to the Housing Demand Survey, the average housing investment proportion in the suburbs from the first quarter of 2010 to the third quarter of 2012 was 15.46%; meanwhile, the proportion of housing investment was 23.12% in the city centre. However, from the fourth quarter of 2012 to the fourth quarter of 2014, the average housing investment proportions were 18.16% and 19.61% in the suburbs and city centre, respectively. Clearly, the proportion of housing investment increased in the suburbs. In addition, the suburbs had a higher vacancy rate, but constructors still applied for construction licenses.[4] The construction licenses were approved for 67,942 houses in the suburbs and 18,275 houses in the city centre in
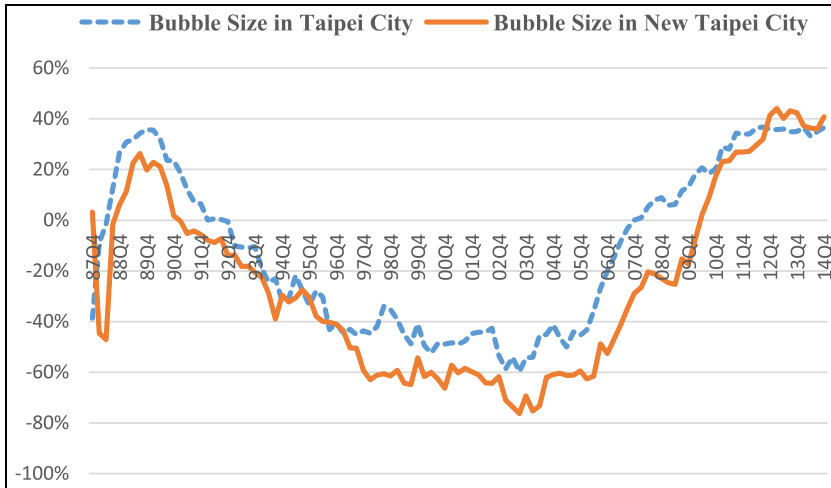
Figure 6. Housing bubble size in Taipei City and New Taipei City.

the recent 3 years. Overall, the suburban housing market exhibits an overinvestment and overdevelopment problem; therefore, the suburbs have a larger bubble than the city centre does. This result is in line with our expectations, according to which the suburbs have a higher vacancy rate, indicating that a large amount of capital is currently invested in vacant real estate.

In addition, Figure 6 illustrates a negative bubble in the markets. According to Shiller (2003), feedback may produce a negative bubble because downward price movement drives further downward price movement, promoting word-of-mouth pessimism, until the housing market reaches an unsustainable low level. When negative bubbles exist in the market, the price reflects the real value of housing.

A housing bubble appeared earlier in city centre than in the suburbs, and the two growth rates are clearly different. Table 4 shows that, when the bubble price in the city centre initially gradually increases, from the first quarter of 2005 to the fourth quarter of 2009, the slope of the city centre bubble size curve is 2.98. In that period, the slope of the

Table 4. Slope of bubble size curve in the Taipei metropolitan area.

	Taipei City (centre)	New Taipei City (suburbs)
2005Q1–2009Q4	2.98	2.15
2010Q1–2014Q4	0.93	2.41

bubble size curve is 2.15 for the suburbs. However, when the housing bubble appears in the suburbs, from the first quarter of 2010 to the fourth quarter of 2014, the slope of the bubble size curve is 2.41 in the suburbs and 0.98 in the city. This implies the presence of substantial speculative behaviour in the suburbs, which causes the bubble to expand rapidly.

Housing price cointegration in city centre and suburbs

Housing prices can be divided into two components: fundamental and bubble prices. However, previous studies have failed to

Table 5. Augmented Dickey–Fuller statistic on Engle-Granger cointegration for housing prices.

	Housing price		Fundamental		Bubble	
	$e^h_{c,t}$	$e^h_{s,t}$	$e^f_{c,t}$	$e^f_{s,t}$	$e^b_{c,t}$	$e^b_{s,t}$
Taipei City (downtown)	–	−2.73*	–	−2.36	–	−3.38**
New Taipei City (suburbs)	−2.16	–	−2.11	–	−3.17**	–

Note: ** and *denote rejection of the null hypothesis at 5% and 10% significance levels, respectively.

Table 6. Error correction of housing prices using Engel-Granger cointegration regression.

$\Delta \mathbf{X} \backslash \Delta \mathbf{Y}$		$\Delta P^h_{c,t}$	$\Delta P^h_{s,t}$	$\Delta P^b_{c,t}$	$\Delta P^b_{s,t}$
Taipei City (downtown)	β_1		0.16***		0.50***
	λ		−0.14***		−0.05***
New Taipei City (suburbs)	β_1	0.50***		0.48***	
	λ	−0.05		−0.02	

Note: β_1 denotes short − run elasticity, λ denotes speed of adjustment parameter to equilibrium. ** and *** denote rejection of the null hypothesis at 5% and 1% significance levels, respectively.

investigate how housing price diffusion is driven by these components. Therefore, in this study, we used the Engle–Granger cointegration and Granger causality tests to analyse links amongst regional housing, fundamental, and bubble prices.

Table 5 shows that the unit root test on residuals obtained from the housing price regression equation is stationary; in other words, Taipei City and New Taipei City housing prices are cointegrated. The results comply with the notion that housing prices diffuse from the city centre (Taipei City) to the surrounding areas (New Taipei City). The unit root test on residuals obtained from the fundamental regression equation is nonstationary, revealing that the fundamental prices of Taipei City and New Taipei City are not cointegrated. We found that the unit root test on residuals obtained from the bubble price regression equation is stationary based on the cointegration regression using a time trend; the Taipei City and New Taipei City bubble prices are cointegrated. Therefore, the results are as predicted, and

bubble prices force housing price diffusion, whereas fundamental prices do not.

After determining that the housing and bubble prices are cointegrated, the regression can be expressed as an error correction model. The symbol β_1 represents short-run elasticity, which shows the short-run change rates. The symbol λ is the speed of adjustment to parameters that represent the rate of departing from a long-run equilibrium to a disequilibrium Table 6 lists the housing price and bubble price dynamic trends, respectively. The evidence indicates that housing prices diffuse from Taipei City to New Taipei City, and the bubble price is the major factor that increases New Taipei City housing prices.

Finally, we use the Granger causality test to analyse relationships amongst the housing, fundamental and bubble prices between the city centre and suburbs. Results of the Granger causality test are shown in Table 7. The hypothesis of non-Granger causality is rejected at a 10% significance level. Our F statistics indicate that housing prices in the

Table 7. Results of Granger causality test.

Null hypothesis	F statistic
Housing price	
City central ($\Delta P_{c,t}^h$) does not Granger Cause Suburban ($\Delta P_{s,t}^h$)	12.52***
Suburban ($\Delta P_{s,t}^h$) does not Granger Cause City Central ($\Delta P_{c,t}^h$)	4.04**
Fundamental	
City central ($\Delta P_{c,t}^f$) does not Granger Cause Suburban ($\Delta P_{s,t}^f$)	3.18***
Suburban ($\Delta P_{s,t}^f$) does not Granger Cause City Central ($\Delta P_{c,t}^f$)	0.45
Bubble price	
City central ($\Delta P_{c,t}^b$) does not Granger Cause Suburban ($\Delta P_{s,t}^b$)	13.76***
Suburban ($\Delta P_{s,t}^b$) does not Granger Cause City Central ($\Delta P_{c,t}^b$)	4.97***

Note: ** and *** denote rejection of the null hypothesis at 5%, and 1% significance levels, respectively.

city centre and suburbs have a bilateral relationship. The price interaction between the two adjacent regions is expected. However, examining the F statistics further reveals that the F statistic associated with changes in the suburbs originating from the city centre is more significant than that associated with changes in the city centre originating from the suburbs. The F statistics imply that the influence of housing prices in the city centre on housing prices in the suburbs is stronger than that of housing prices in the suburbs on prices in the city centre. The causal relationship between fundamental prices in the two areas functions in only one direction: changes in the city centre Granger cause changes in the suburbs. This implies that housing in the city centre is excessively expensive for many consumers; therefore, consumers have moved to the suburbs to buy housing. Taipei City has an extremely limited area and is consistently the starting point for housing price booms in Taiwan.

Bubble prices in the city centre and suburbs have a bilateral relationship. Because investors seek maximum returns, housing investment in peripheral markets is a substitute for investment in the urban core market. Therefore, many investors are attracted to invest in the suburbs, implying that investor activities migrate from the city centre to the suburbs. This result indicates that investor activities move throughout the city and the suburbs, causing the bubbles to have two-way causality. Examining the F statistics further reveals that the F statistic associated with changes in the suburbs originating from the city centre is more significant than that associated with changes in the city centre originating from the suburbs. The F statistics imply that the influence of bubble prices in the city centre on bubble prices in the suburbs is stronger than that of bubble prices in the suburbs on bubble prices in the city centre.

In summary, bubbles in the city centre and suburbs can be estimated with statistical significance by using our state-space model. Through the Engle–Granger cointegration and Granger causality tests, we found that housing prices in the city centre diffuse to the suburbs through the bubble contagion. When housing prices diffuse from the city centre to the suburbs, the real city centre prices are higher than those in the suburbs; however, when the bubble price diffuses from the city centre to the suburbs, the bubble in the suburbs is larger than that in the city centre.

Conclusion

Previous studies have indicated that housing prices diffuse from central to surrounding areas. However, most studies have ignored

the notion of housing prices comprising fundamental and bubble prices. This paper focuses on the hypothesis that housing price diffusion may be caused by the bubble price. We argue that the investment cost in suburbs is lower than that in the city centre for obtaining high profits, and thus, investors shift their investment targets from the city centre to suburbs. Therefore, when speculative behaviour increases in suburbs, the housing bubble expands rapidly. The housing bubble in suburbs is larger than that in the city centre. This paper is the first to directly distinguish how overall housing, fundamental, and bubble prices in the city centre and suburbs affect each other. We clarify how bubble prices force housing price diffusion in the Taipei metropolitan area.

This study yielded two valuable contributions. First, our empirical results show that housing bubbles existed in the housing markets of Taipei City and New Taipei City in 2014. The Engle–Granger cointegration test results indicate that the housing and bubble prices of the two cities are cointegrated, but fundamental prices are not. Furthermore, the Granger causality test results show that the Granger causality of bubble prices (changes in the city centre Granger cause changes in the suburbs) is more significant and powerful than the fundamental. Therefore, the housing bubble appears to have a contagion effect, and bubble prices force housing prices to increase in New Taipei City, causing housing price diffusion from the city centre to the suburbs. These findings explain that the bubble price is the main factor causing the spread of housing prices, differing from findings reported in general studies.

Second, Fernández-Kranz and Hon (2006) suggested that a price boom caused by an abnormal demand shock qualifies as a bubble. From the fourth quarter of 2012 to the fourth quarter of 2014, the housing bubble size in suburbs was larger than that in city centre, which is an unprecedented

phenomenon, indicating that there is more pseudo demand in the suburbs. The difference could be due to the overconfidence of investors overinvesting in suburban areas, which has caused further overinvestment by real estate developers. As we can see that the vacancy rate reached 22%, with approximately 328,742 vacant homes in New Taipei City. The New Taipei City vacancy rate rose by 4.75% over the decade. The housing market in New Taipei City is substantially affected by speculation. The vacancy rate for Taipei City reached 12%, with approximately 122,905 vacant homes. In other words, many areas have gradually become ghost towns. This phenomenon has led to inadequate urban development. Because the origin of a city is from the gathering of humans, the number of houses increasing without human aggregation in a city indicates that the city lacks development potential. Furthermore, although the vacancy rate in suburbs was higher than that in the centre, however, constructors continue to apply for construction licenses. Construction licenses[5] have been approved for 67,942 houses in the suburbs and 18,275 houses in the city centre in the recent 3 years. The suburban housing market has experienced an oversupply problem. These overinvestments increased the risk for both investors and the industry. Some investors may have realised this risk, prompting them to continue seeking opportunities for arbitrage profits in other areas, which has further caused the bubble to have a contagion effect in other areas. As since 2003, the housing price bubbles that initiated from the Taipei area have spread to other parts of Taiwan. Our study demonstrates that when housing bubbles force housing price diffusion, the bubble in the suburbs is larger than that in the city centre. To prevent the financial crisis because of the housing bubble from bursting, the central bank should implement measures that are more stringent in suburbs such as raising the

mortgage rate, implementing loan controls to reduce the money supply, and limiting the amount for construction loans.

In summary, this investigation of the housing bubble contagion in the Taipei metropolitan area demonstrates that both housing and bubble prices diffused from the city centre to the suburbs, although suburban housing prices were lower than those of the city centre; however, the suburban housing bubble was larger than that of the city centre. This implies that housing prices in the suburbs are more inflated, and speculative behaviour is prominent in the suburban housing market. Therefore, authorities must address the problem of high housing prices to prevent this bubble from bursting and reduce speculation in the housing market. In addition, Acemoglu (1995) noted that an increase in rent seeking reduces the returns on both entrepreneurship and rent seeking. Therefore, rent seeking is harmful to housing markets. Investors, in particular, purchase housing not to live in, but to wait for a favorable opportunity to sell at higher prices, causing the vacancy rate to be high in the suburban housing market, which is unfavorable for the development of the housing market and urban growth. The method used for estimating bubble prices in this study may also be used to monitor the health of housing markets and assist policymakers in making more informed decisions.

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Notes

1. Abraham and Hendershott (1996) indicated that housing price appreciation is determined by the equilibrium price and dynamic adjustment. Maurice (2001) stated that the asset price can be decomposed into two components, fundamental and non-fundamental prices. Both of these concepts are equivalent to the notion of asset prices being composed of fundamental and bubble prices (Blanchard and Fisher, 1989).
2. http://win.dgbas.gov.tw/fies/.
3. Hendry (1984) used the Almon polynomial (see Sargan, 1980). The definition is

$$A_n(Y_t) = \frac{2}{(n+1)} \sum_{i=0}^{n} (n-i)Y_{t-i} \quad \text{for} \quad t = 1,2....n,$$

   where a ($\bullet$) is a restricted Almon polynomial, Y_t is the current income, and (n-i) is the selected calculation period. Here, we set $n = 3$, and I should be lower than 3; therefore, $i = 0,1,2$.
4. http://sowf.moi.gov.tw/stat/month/list.htm.
5. http://sowf.moi.gov.tw/stat/month/list.htm.

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Appendix

The fundamental house price–income model

The model described in the Methodology has the following process:

$$pq_t = k + \mu pq_{t+1} + \Delta q_{t+1} - r_{t+1} \quad (1)$$

After repeated substitution in Equation (1), we obtain:

$$pq_t = \frac{k(1-\mu^i)}{(1-\mu)} + \sum_{j=0}^{i-1} \mu^{j+1} \Delta q_{t+j+1} \\ - \sum_{j=0}^{i-1} \mu^{j+1} r_{t+j+1} + \mu^i pq_{t+1} \quad (2)$$

Letting $t \to \infty$, and assuming that the limit of the final term is 0, Equation (2) can be rewritten as:

$$pq_t = \frac{k}{(1-\mu)} + \sum_{j=0}^{\infty} \mu^{j+1} \Delta q_{t+j+1} \\ - \sum_{j=0}^{\infty} \mu^{j+1} r_{t+j+1} \quad (3)$$

If $q_t \sim I(1)$, then $\Delta q_t \sim I(0)$, and, assuming that $r_t \sim I(0)$ (recall that it is the real discount rate), pq_t is $I(0)$. Taking the conditional expectations of both sides yields:

$$pq_t = \frac{k}{(1-\mu)} + \sum_{j=0}^{\infty} \mu^{j+1} E_t \Delta q_{t+j+1} \\ - \sum_{j=0}^{\infty} \mu^{j+1} E_t r_{t+j+1} \quad (4)$$

where $E_t r_{t+j+1}$ is investors' required return.

In order to use Equation (4) to generate a series for pq_t^* and p_t^*, a three-variable VAR model ($pq_t, \Delta q_t \text{ and } \sigma_t^2$) is used to forecast real permanent income growth and housing return variance, and the ratio of the fundamental housing price to permanent income, pq_t^*, is measured. The procedure of Merton (1973) is applied to the intertemporal CAPM; α is the time-varying risk premium as the product of the coefficient of relative risk aversion, and $E_t \sigma_t^2$ is the expected variance of returns. The equation for the price-income ratio then becomes:

$$pq_t = \frac{k-f}{(1-\mu)} + \sum_{j=0}^{\infty} \mu^{j+1} E_t \Delta q_{t+j+1} \\ - \alpha \sum_{j=0}^{\infty} \mu^{j+1} E_t \sigma_{t+j+1}^2 \quad (5)$$

where f is the constant real-risk-free component of real required returns.